# Technical Note

## 1971-25

A. R. Dion

## Performance of a 110-Wavelength EHF Waveguide Lens

23 April 1971

# Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

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LINCOLN LABORATORY

# PERFORMANCE OF A 110-WAVELENGTH
# EHF WAVEGUIDE LENS

*A. R. DION*

*Group 61*

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# ABSTRACT

Expanded aluminum honeycomb with a nominal cell width of 0.156 inch was used to fabricate a 24-inch-diameter unzoned lens for operation at 55 GHz. The lens exhibits good focusing properties but its efficiency of only 18 percent reflects significant random phase errors. These errors are shown to result mainly from variations of the cell width. From the measured radiation characteristics the average width of the cells is deduced to be 0.161 inch with a standard deviation of 0.003 inch. Since phase errors associated with variations of the cell width are proportional to the lens thickness it is deduced that the much thinner zoned lenses fabricated from this material would exhibit efficiencies approaching 50 percent.

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# Performance of a 110-Wavelength EHF Waveguide Lens

## INTRODUCTION

A 24-inch EHF waveguide lens utilizing aluminum honeycomb as the guiding medium was built and tested to determine the properties of this material as lens media. Expanded commercial honeycomb with a 5/32-inch mesh size was used to fabricate the plano-concave lens for operation at a center frequency of 55 GHz. To construct the lens a 24-inch right-cylinder slab was cut out from a rectangular honeycomb slab about 10-inch thick. The cylinder slab was epoxy-bonded to a ring for support and potted with a low-melting-point compound to enable machining of the concave surface and facing off of the flat surface. After melting out the potting material the lens shown in Fig. 1 was obtained. This memorandum describes the principal design considerations and presents the results of the performance tests.

## DESIGN CONSIDERATIONS

The aluminum honeycomb material is fairly uniform; examination shows cells approximating a regular hexagon with deviations from one another of sufficient magnitude, however, to be observed visually. The cell size is specified by the distance across flat surfaces as depicted in Fig. 2 and is nominally equal to 0.156 inch. Since the wavelength in the honeycomb waveguides is greater than the free-space wavelength, the medium behaves like a refracting medium with index of refraction less than unity. This index is
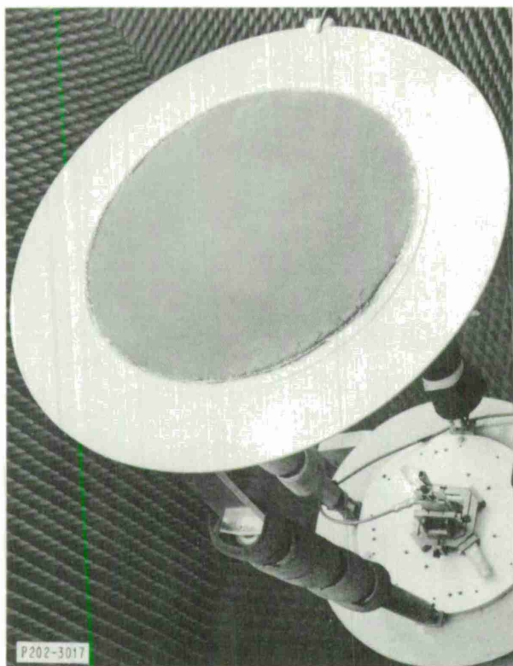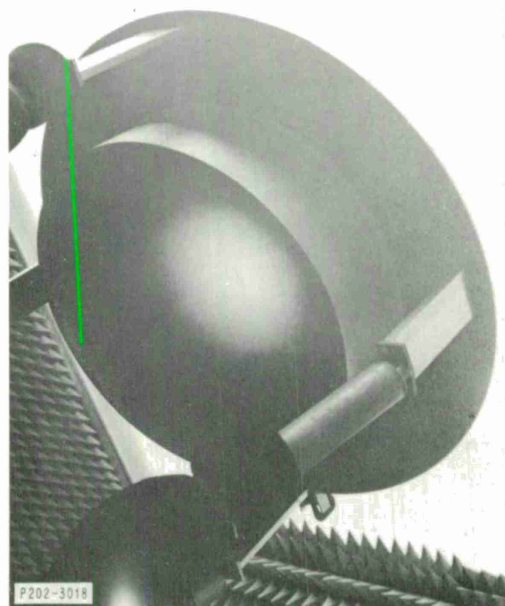
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Fig. 1(a). Front view of EHF lens.



Fig. 1(b). Rear view of EHF lens.

$$n = \left[1 - (\lambda/\lambda_c)^2\right]^{1/2} \tag{1}$$

with $\lambda_c$, the cut-off wavelength given by [1]

$$\lambda_c = 1.792\,a \tag{2}$$

where a is the distance across the flat surfaces of the hexagonal waveguide.

Substituting a = 0.156 inch and $\lambda$ = 0.2147 inch yields n = 0.642 at a frequency

of 55 GHz. The plano-concave geometry was chosen to simplify construction.

The shape of the concave surface is determined by applying the condition of

equality of optical paths. Referring to the coordinate system of Fig. 3 and

equating the optical path of a general ray to that of the central ray gives

$$\left[(x + F)^2 + y^2\right]^{1/2} - nx = F \tag{3}$$

which may be transformed to the form

$$\left[\frac{x + F/(1 + n)}{F/(1 + n)}\right]^2 + \frac{y^2}{F^2(1 - n)/(1 + n)} = 1 \tag{4}$$

i. e. , the equation of an ellipse with the feed at the focus farther from the

origin. Since the maximum diameter of the lens is equal to the minor axis of

the ellipse the minimum ratio of focal length to diameter is

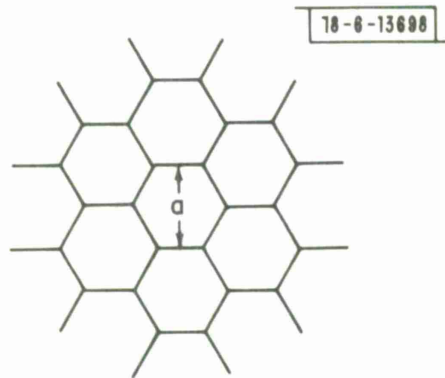$$\left[F/D\right]_{min} = \frac{1}{2}\sqrt{\frac{1 + n}{1 - n}} \tag{5}$$
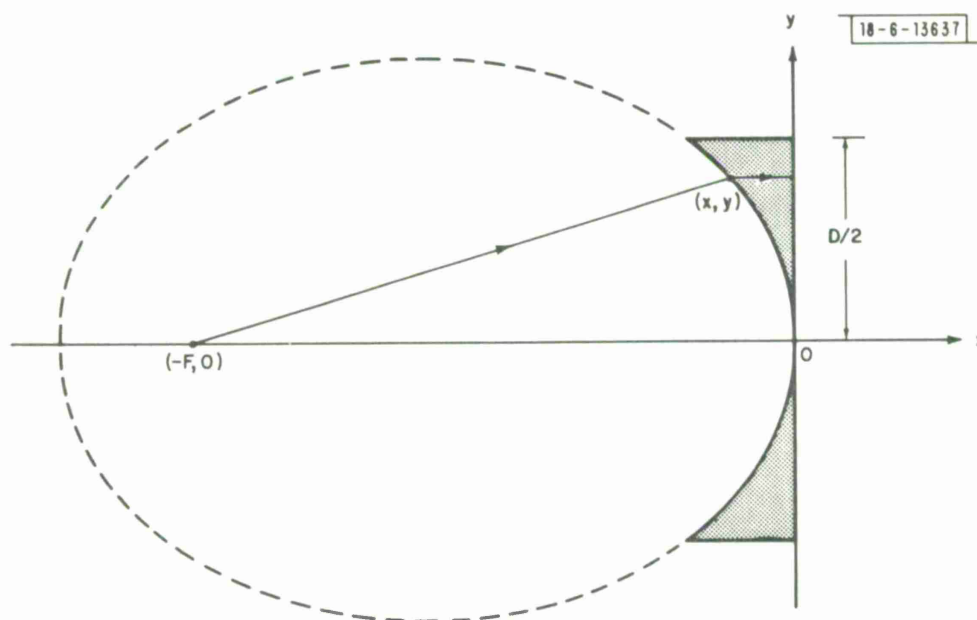
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Fig. 2. Honeycomb cell geometry.



Fig. 3. Coordinate system.

which is equal to 1.07 for n = 0.642. A design with a minimum $F/D$ ratio would be excessively thick, would lead to large incidence angles on the concave surface and is to be avoided. An $F/D$ ratio of 1.5 was chosen making the focal length equal to 36 inches, the angular aperture, 45.2 degrees, and the maximum thickness, 6.679 inches.

The focal length of the lens is a function of frequency since its index of refraction is frequency dependent. The focal length dependence on index of refraction is identical to that of thin optical lenses [2] i.e.,

$$F = F_o (1 - n_o)/(1 - n) \tag{6}$$

where $F_o$ is the focal length corresponding to the design index of refraction $n_o$, and n is as given by Eq. (1).

The characteristics of the lens have been studied with a linearly polarized feed providing a 10-dB taper of amplitude in both the E- and H-planes. The feed (Fig. 4) is a pyramidal horn with a rectangular aperture excited by a $TE_{10}$ mode. The amplitude pattern of this feed in conventional spherical coordinates is

$$F(\theta, \varphi) = \frac{1 + \cos \theta}{2} \cdot \frac{\sin u_1}{u_1} \cdot \frac{\cos u_2}{1 - (2u_2/\pi)^2} \tag{7}$$

where
$$u_1 = \pi d_1 \sin \theta \cos \varphi / \lambda$$

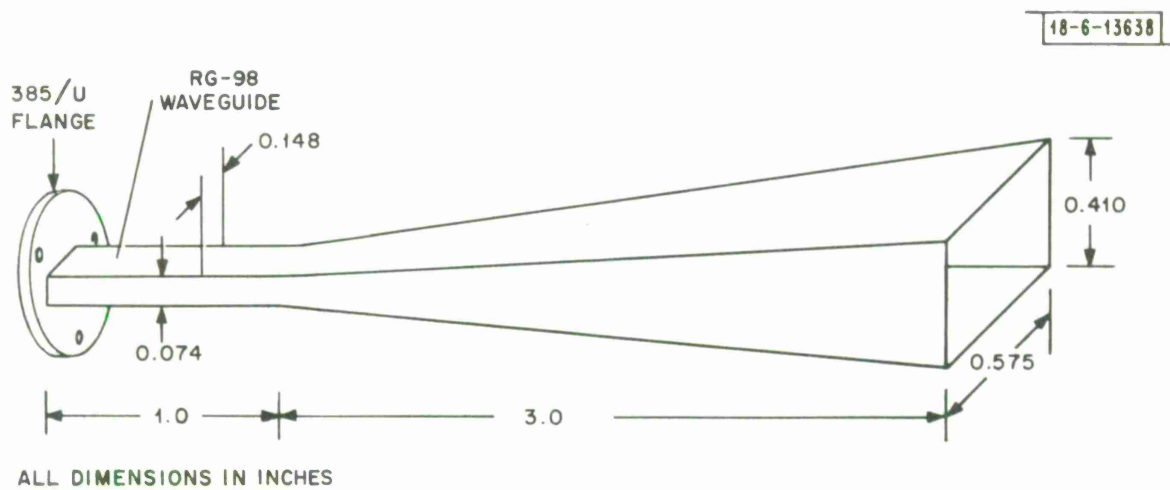$$u_2 = \pi d_2 \sin \theta \sin \varphi / \lambda$$

Fig. 4.  Feed horn.

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and $d_1$ and $d_2$ are the aperture dimensions in the E- and H-plane, respectively.

Equation (7) is quite precise as evidenced in Figs. 5 and 6 where the measured and computed principal plane patterns of the horn are compared. The gain of the feed horn is 17.1 dB.

LENS PERFORMANCE

Because the honeycomb cell geometry deviates somewhat from a perfect regular hexagon the behavior of the lens may be expected to be a function of the orientation of the polarization vector with respect to the cell walls. For this reason, the lens performance was studied for two cases of polarization; in the first case the polarization vector was parallel to the flat faces of the cell and in the second case, perpendicular.

Radiation patterns measured with the polarization vector parallel to the flats are presented in Figs. 7 and 8. Also shown in these figures are the calculated radiation patterns of the error-free lens. Good focusing properties are demonstrated by the well-defined beam and the relatively small side lobes. The measured focal length of 37.6 inch deviates by 4.5 percent from the design value. The half-power beamwidth of 0.7° versus an anticipated value of 0.6° is indicative of greater aperture taper than designed for. The additional tapering results principally from energy being transferred to the orthogonal polarization due to some skewing of the honeycomb cells and to energy lost to diffracted waves due to the grating effect. Evidence of substantial depolarization is given in Fig. 9 where the radiation pattern of the cross-polarized
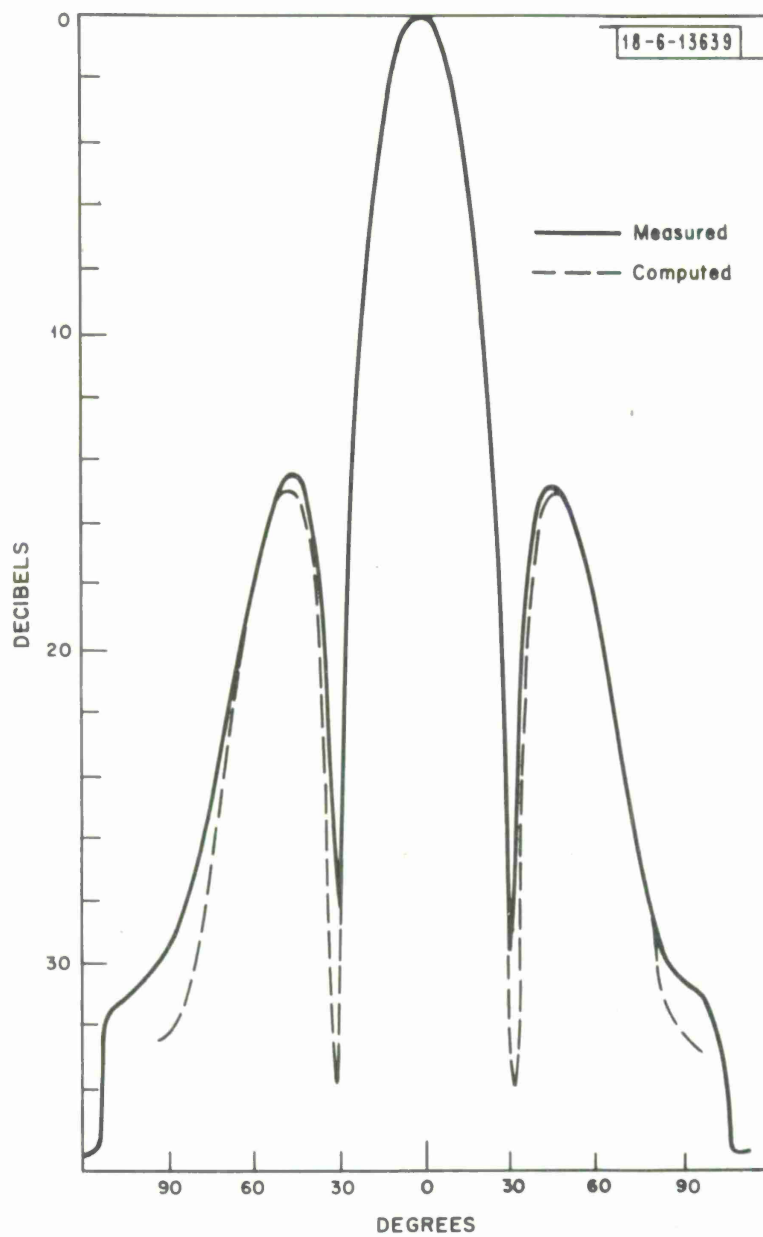
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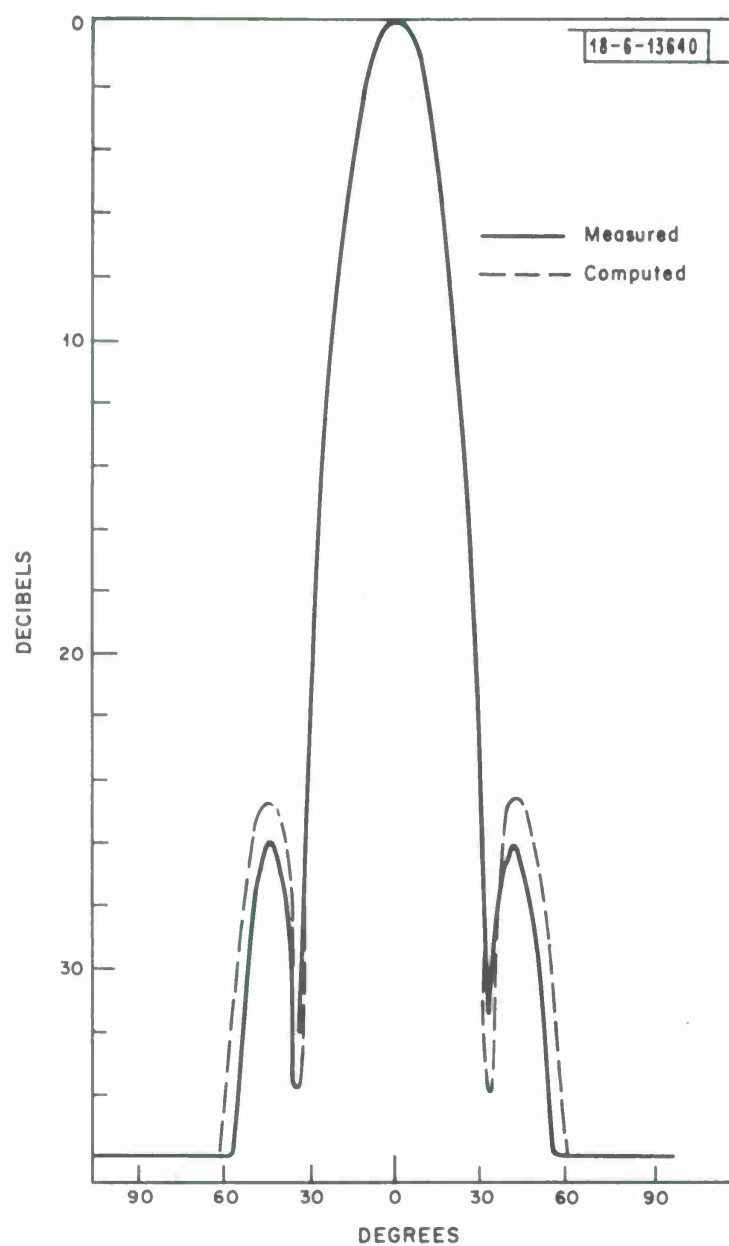Fig. 5. E-plane pattern of feed horn.
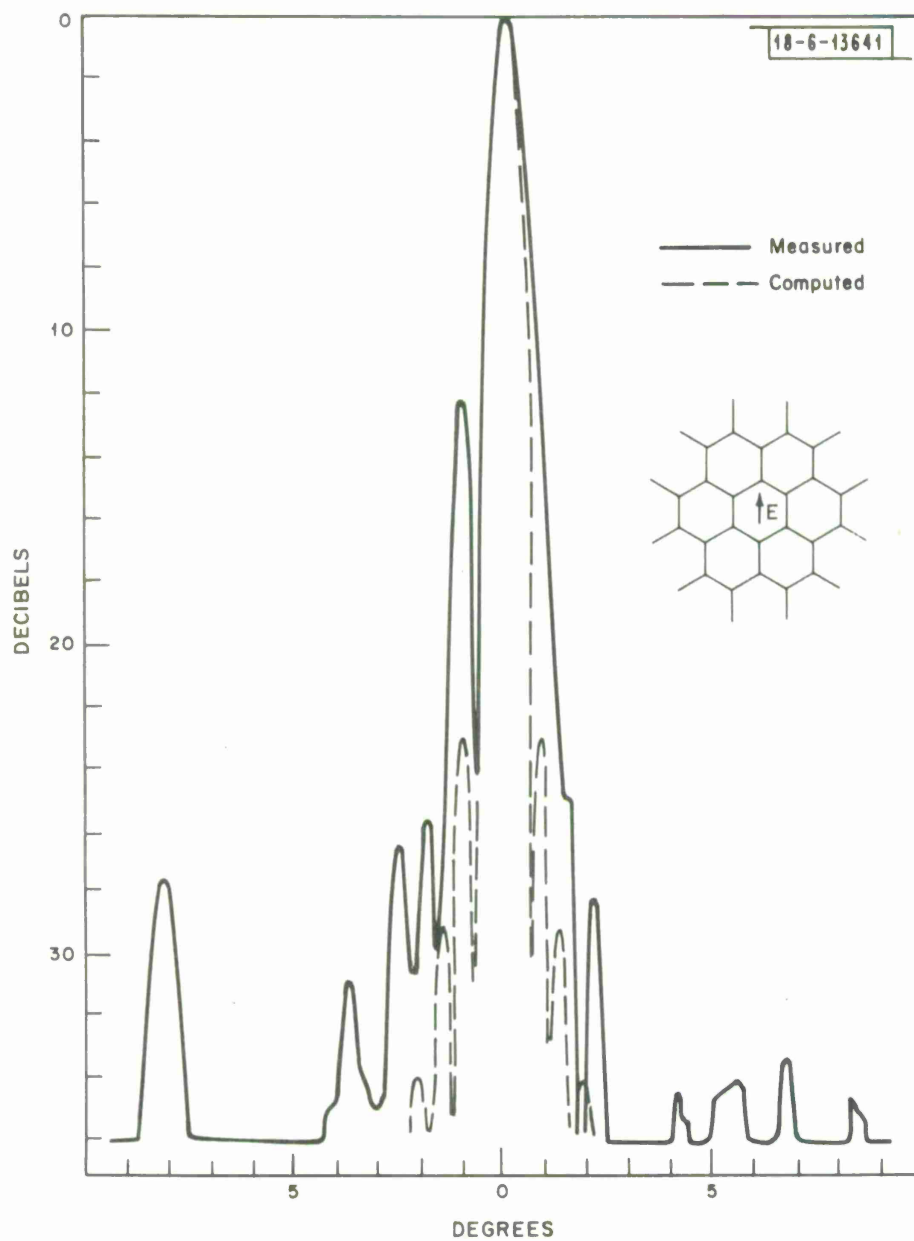
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Fig. 6.   H-plane pattern of feed horn.

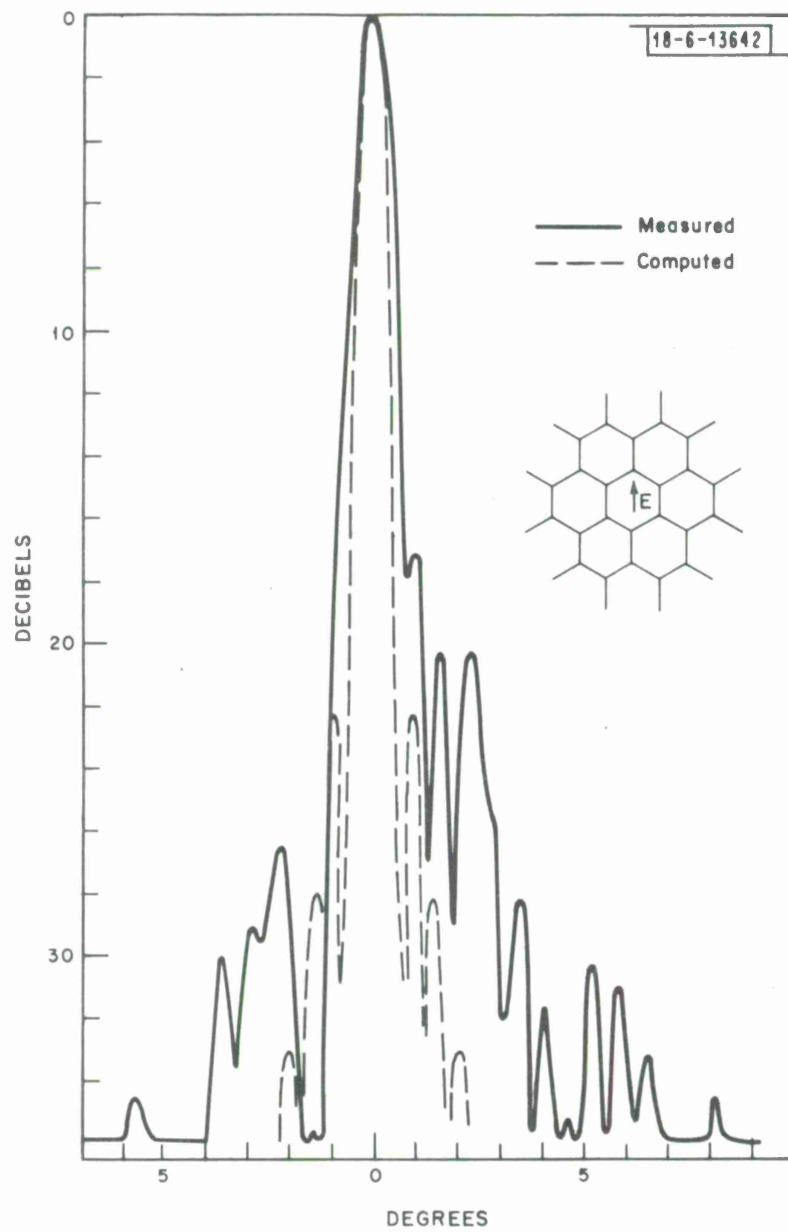Fig. 7. E-plane pattern of EHF lens antenna.
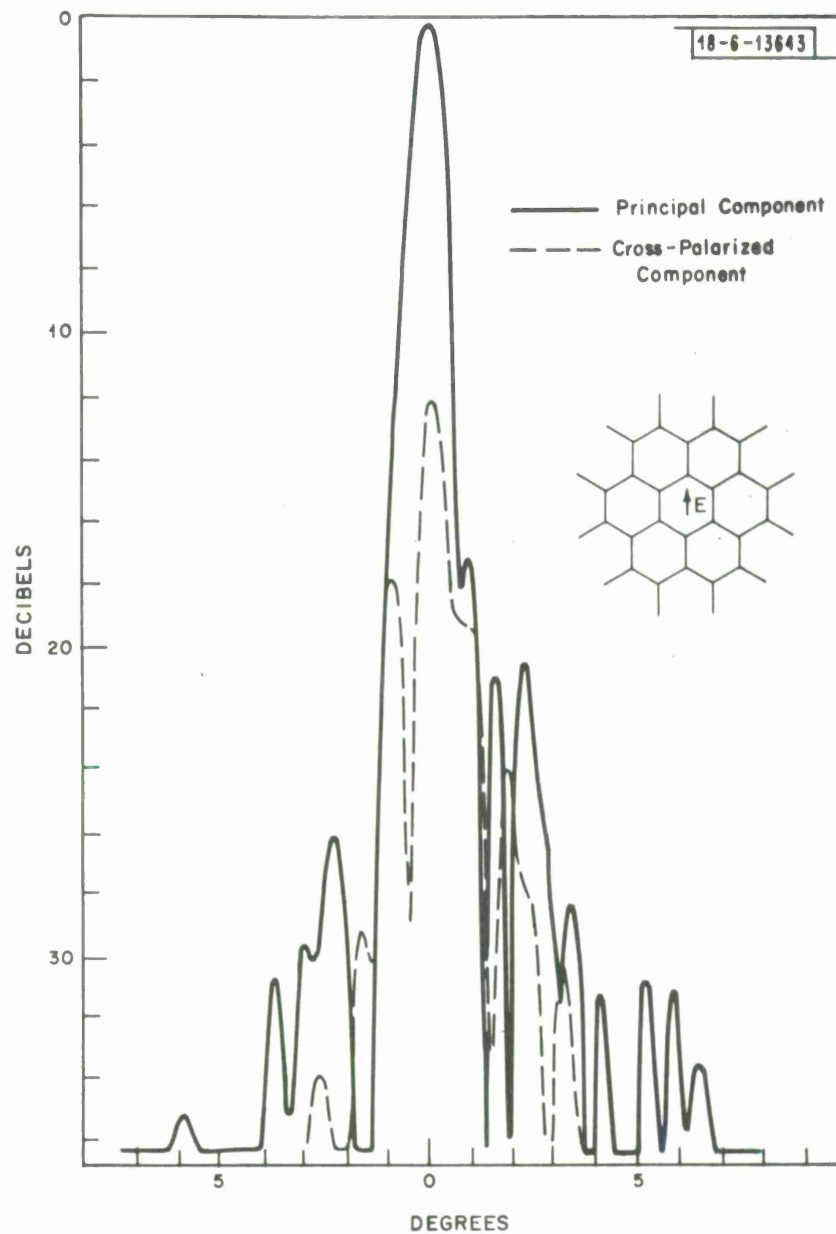
Fig. 8. H-plane pattern of EHF lens antenna.

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Fig. 9. Superimposed principal and cross-polarized H-plane pattern.

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component is compared to that of the principal component. The grating waves are due to the curvature of the lens which increases the spacing between the radiation centers of the cells sufficiently to allow these waves to exist [3]. Calculations indicate that about one half the power in the marginal rays is diverted into grating waves.

Radiation patterns measured with the polarization vector perpendicular to the flat surfaces of the honeycomb cell show generally the same features as the previous ones. However, the focal length (40.2 inches) differed from the previous case indicating a polarization dependence of this latter.

Thus the focal length averages 38.9 in. and varies by ± 4 percent with polarization direction. From the average focal length the mean distance between the flats of the cells may be obtained. Differentiating Eqs. (1), (2) and (6) one obtains

$$\frac{da}{a} = \frac{d\lambda_c}{\lambda_c} = \frac{n}{1 - n^2}\, dn \tag{8}$$

$$\frac{dF}{F} = \frac{dn}{1 - n} \tag{9}$$

$$da = \frac{n}{1 + n}\, \frac{dF}{F}\, a = 0.005 \text{ inch} \quad . \tag{10}$$

Thus the average cell is 0.005 inch wider than the nominal value.

The most significant parameter of the lens antenna is its gain. This latter, measured by comparison to a standard gain horn, is 43.5 ± 0.3 dB at a

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frequency of 55 GHz corresponding to an efficiency of 18.3 percent. This low efficiency indicates considerable loss due to random variations of phase across the aperture. Furthermore, since the lens focalizes radiation in a narrow beam with low side lobes, the energy loss through random phase deviations apparently is scattered in a wide pattern indicating that the correlation interval between random errors is small and, most likely, is equal to the mesh size. Thus the principal contributor to the phase errors are the random variations of index of refraction i.e., random variations of the width of each hexagonal waveguide. In addition to random phase errors other significant contributors to the gain loss are surface reflections, depolarization and grating waves. The fraction of power reflected back at each surface is given by $(n-1)^2/(n+1)^2$ and amount to a loss of gain of about 0.5 dB for both surfaces. The depolarization and grating wave losses have been roughly estimated to be 1 dB and 0.5 dB, respectively. Thus the loss due to effects other than random phase errors is equal to about 2.0 dB. The calculated gain of the error-free lens is 49.4 dB and therefore a loss of 5.9 dB has to be accounted for. Deducting the surface reflection, depolarization and grating wave losses from this figure leaves 3.9 dB to be attributed to random phase errors. From the statistical analysis of random phase errors [4] the corresponding standard deviation of optical path is 0.032 inch. This deviation is the sum of the standard deviation due to surface profile errors and to index of refraction errors. The lens surfaces were machined to an rms value of 0.005 inch to which

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corresponds a standard deviation of optical path equal to $0.01 (1 - n) = 0.0036$ inch. Thus the standard deviation of optical path due to the random variations of the index of refraction only is $\sigma \approx 0.028$ inch. This standard deviation is a weighted average because the aperture illumination is not uniform. From the measured beamwidth of the secondary pattern the distribution of amplitude over the aperture is deduced to be

$$f(r) = \left(1 - r^2\right)^2 \tag{11}$$

where r is the distance of a honeycomb cell to the lens axis normalized to the lens radius. Since

$$\sigma = \sigma_n L \tag{12}$$

where $\sigma_n$ is the standard deviation of index of refraction and L is the length of a honeycomb cell which from Eq. (3) may be approximated by

$$L \approx \frac{r^2 D^2}{8(1 - n) F} \tag{13}$$

the standard deviation of optical path becomes

$$\sigma = \frac{\sigma_n D^2}{8(1 - n) F} \frac{\int_o^1 \left(1 - r^2\right)^2 r^3 \, dr}{\int_o^1 \left(1 - r^2\right)^2 r \, dr} \tag{14}$$

from which results

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$$\sigma_n = 32(1-n)\frac{F}{D^2}\sigma \quad . \tag{15}$$

Associating the standard deviations to the differentials as given by Eq. (8), we have

$$\sigma_n = \frac{1-n^2}{n}\frac{\sigma_a}{a} \tag{16}$$

and, therefore, $\sigma_a$ the standard deviation of the mesh width is

$$\sigma_a = 32\frac{n(F/D)}{(1+n)(D/a)}\sigma \simeq 0.003 \text{ inch} \quad . \tag{17}$$

Thus the honeycomb cell has an average width of 0.161 inch with a standard deviation of 0.003 inch.

DISCUSSION

The gain loss results principally from random phase errors due to variations of the index of refraction. The amplitude of these errors is proportional to the thickness of the lens and, therefore, would be reduced considerably by zoning the lens. At the same time, depolarization and focal length variations which are also functions of the lens thickness would be considerably reduced. The thickness of a zoned lens need not be much larger than a full-wave-plate thickness $\approx 0.6$ inch $(\lambda/(1-n))$ corresponding to about ten zones. The gain improvement should be about 5 dB minus about 1 dB due to shadowing by the steps resulting in an effective gain increase of $\approx 4$ dB (losses due to surface reflections and grating waves would reamin). The efficiency of the zoned lens

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would be about 46 percent. To provide a more sturdy lens fewer zones could be made and still reasonable efficiency achieved. For instance with only three zones the estimated efficiency is $\approx$ 40 percent. In addition, zoning the lens will reduce the cross-polarized beam to more than 20 dB below the peak of the principal beam and will reduce the variation of focal length to a negligible amount.

The bandwidth of the lens may be derived from Silver [5]. The bandwidth corresponding to a loss of gain of 0.3 dB is, for the unzoned lens

$$BW = 25 \, n\lambda/(1 - n^2)t \tag{18}$$

where t is the maximum thickness. Substitution of the lens parameters yields a bandwidth equal to 0.6 percent. For the zoned lens this bandwidth is

$$BW \approx 25n/(1 + Kn) \tag{19}$$

where K the number of zones is ten yielding a bandwidth of 2.2 percent.

CONCLUSION

Expanded aluminum honeycomb is a suitable material for the construction of EHF waveguide lenses. A 24-inch-diameter unzoned lens built for operation at 55 GHz provided good radiation patterns but an efficiency of only 18.3 percent. From the radiation characteristics the width of the honeycomb cell was deduced to average 0.161 inch with a standard deviation of 0.003 inch. The low efficiency was shown to be mostly due to the large amplitude of the random

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phase errors associated with unzoned lenses. With the much thinner zoned lenses efficiencies approaching 50 percent appear realizable. Also somewhat higher efficiencies could be expected from double-concave lenses as compared to plano-concave lenses, because the former lenses experience smaller curvature and therefore should exhibit less power lost to grating waves.

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REFERENCES

[1] R. N. Assaly, "The Guide Wavelength of Cylindrical Waveguide of Arbitrary Cross Section and a Computer Program for Calculating Values," Technical Report 313, Lincoln Laboratory, M.I.T. (May 1963).

[2] M. Born and E. Wolf, Principles of Optics (Pergamon Press, Oxford, 1959) p. 162.

[3] H. Jasik, Antenna Engineering Handbook (McGraw-Hill, New York, 1961) pp. 14-33 to 14-36.

[4] J. Ruze, "Antenna Tolerance Theory — A Review," Proc. IEEE, 54, (1966) pp. 633-640.

[5] S. Silver, Microwave Antenna Theory and Design, Radiation Lab Series, 12 (McGraw-Hill, New York, 1948) pp. 408-410.

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